CHAPTER 2   LITERATURE REVIEW

2.1 HEC-RAS

HEC-RAS is one of the most widespread models used to calculate water-surface profiles and energy grade lines in 1-D, steady-state, gradually-varied flow analysis. In 1-D, steady-state, gradually-varied flow analysis, the following assumptions are made:

1. Dominant velocity is in the flow direction;

2. Hydraulic characteristics of flow remain constant for the time interval under consideration; and

3. Streamlines are practically parallel and, therefore, hydrostatic pressure distribution prevails over channel section (Chow, 1959).

Equations illustrating the stated assumptions are discussed in Section 2.2.

2.2 FUNDAMENTAL HYDRAULIC EQUATIONS

Fundamental hydraulic equations that govern 1-D, steady-state, gradually-varied flow analysis include the continuity equation, energy equation, and flow resistance equation. These equations, in addition to the Froude number and other important hydraulic concepts, are noted in the succeeding sections.

2.2.1 Continuity Equation

The continuity equation describes a discharge as constant and continuous over the period of time in consideration (Chow, 1959). The concept of continuity is shown in Equation 2.1:

$$Q = \bar{v}_1 A_1 = \bar{v}_2 A_2$$ **Equation 2.1**

where:

A_1 = cross-sectional area normal to the direction of flow at the downstream cross section (ft^2);

A_2 = cross-sectional area normal to the direction of flow at the upstream cross section (ft^2);

Q = discharge (cfs);

$\bar{v}_1$ = average velocity at the downstream cross section (ft/s); and

$\bar{v}_2$ = average velocity at the upstream cross section (ft/s).

Using the continuity equation, the average velocity is expressed in terms of discharge and cross-sectional area, which is shown in Equation 2.2:

$$\bar{v} = \frac{Q}{A}$$ **Equation 2.2**

where:

A = cross-sectional area normal to the direction of flow (ft^2);

Q = discharge (cfs); and

$\bar{v}$ = average velocity (ft/s).

2.2.2 Energy Equation

Total energy at any point along an open-channel system can be defined as total head in feet of water (Chow, 1959). Total head of water is calculated using the energy equation. The energy equation is used to calculate the total head of water as the summation of the bed elevation, average flow depth, and the velocity head at a cross section, which is illustrated in Equation 2.3:

$$H = z + y + \frac{\alpha \bar{v}^2}{2g}$$
<div align="right">**Equation 2.3**</div>

where:

α = kinetic energy correction coefficient;

g = acceleration of gravity (ft/s^2);

H = total head of water (ft);

$\bar{v}$ = average velocity at a cross section (ft/s);

y = flow depth at a cross section (ft); and

z = bed elevation at a cross section (ft).

The kinetic energy correction coefficient is multiplied by the velocity head to better estimate the velocity head at a cross section. True velocity head at a cross section is generally higher than the estimated velocity head using the average velocity at a cross section. Kinetic energy correction coefficient aids in correcting the difference where values typically range between 1.03 and 1.36 for fairly straight, prismatic channels (Chow, 1959).

2.2.3 Flow Resistance Equation

The flow resistance equation uses a form of Manning's equation to define an equation that applies average roughness to the wetted perimeter of a cross section (United States Army Corps of Engineers (USACE), 2001a). The flow resistance equation is shown in Equation 2.4 based on a form of Manning's equation:

$$Q = KS_f^{1/2}$$ **Equation 2.4**

where:

K = channel conveyance (ft);

Q = discharge (cfs); and

S_f = friction slope (ft/ft).

Conveyance at a cross section is obtained using Equation 2.5:

$$K = \frac{\Phi}{n} AR^{2/3} = \frac{\Phi}{n} A\left(\frac{A}{P}\right)^{2/3}$$ **Equation 2.5**

where:

A = cross-sectional area normal to the direction of flow (ft^2);

Φ = unit conversion (Eng = 1.486 and SI = 1.000);

K = channel conveyance (ft);

n = roughness coefficient;

P = wetted perimeter (ft); and

R = hydraulic radius (ft).

Cross-sectional area and wetted perimeter are a function of channel geometry. If the cross section is rectangular, then Equation 2.6 and Equation 2.7 apply for cross-sectional area and wetted perimeter, respectively:

$$A = yw$$ **Equation 2.6**

$$P = 2y + w$$ **Equation 2.7**

where:

A = cross-sectional area normal to the direction of flow (ft^2);

P = wetted perimeter (ft);

w = top width of a cross section along the water surface (ft); and

y = flow depth at a cross section (ft).

Figure 2.1 illustrates the variables used in Equation 2.6 and Equation 2.7 for a rectangular cross section.
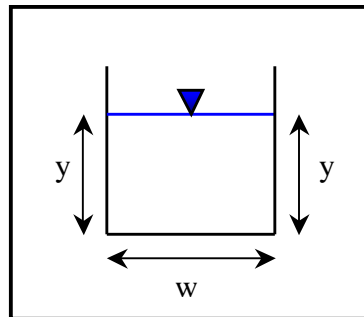

Figure 2.1. Variables Used to Calculate A and P

2.2.4 Energy Loss in an Open-channel System

Energy loss in an open channel system is defined as energy loss along a channel reach due to friction, contractions, expansions, eddies, spiral, and secondary currents. In 1-D, steady-state, gradually-varied flow analysis, energy loss is assumed to be due to friction, contraction, and expansion loss.

2.2.4.1 Friction Loss

Friction loss is termed as energy loss along a channel reach due to roughness of the channel boundary. Friction loss is calculated by multiplying average friction slope by the distance along the channel. Equation 2.8 illustrates the friction loss equation:

$$h_f = \overline{S}_f \Delta x \qquad\qquad \textbf{Equation 2.8}$$

where:

h_f = energy loss due to friction (ft);

$\overline{S}_f$ = average friction slope between two adjacent cross sections (ft/ft); and

Δx = incremental channel length (ft).

Average friction slope is calculated by rearranging Equation 2.4. Equation 2.9 presents the equation for average friction slope:

$$S_f = \left(\frac{Q}{K}\right)^2 \qquad\qquad \textbf{Equation 2.9}$$

where:

K = channel conveyance (ft);

Q = discharge (cfs); and

S_f = friction slope (ft/ft).

A statistical technique known as the average conveyance method is used to calculate the average friction slope between adjacent cross sections. The average conveyance method is illustrated by Equation 2.10:

$$\overline{S}_f = \left(\frac{Q_1 + Q_2}{K_1 + K_2}\right)^2 \qquad\qquad \textbf{Equation 2.10}$$

where:

K_1 = channel conveyance at the downstream cross section (ft);

K_2 = channel conveyance at the upstream cross section (ft);

Q_1 = discharge at the downstream cross section (cfs);

Q_2 = discharge at the upstream cross section (cfs); and

$\overline{S}_f$ = average friction slope between two adjacent cross sections (ft/ft).

Average conveyance method is the default method in HEC-RAS to calculate average friction slope (USACE, 2001a).

2.2.4.2 Minor Loss

Expansion and contraction losses are collectively known as minor loss along a reach in a 1-D, steady-state, gradually-varied flow analysis. Expansion and contraction minor loss is related to the energy loss due to changes in cross-sectional shape along the reach. For instance, when water flows downstream, a reach may expand or contract. As the reach expands or contracts, energy loss occurs along a study reach. Figure 2.2 illustrates a planform view of a contraction reach and an expansion reach.
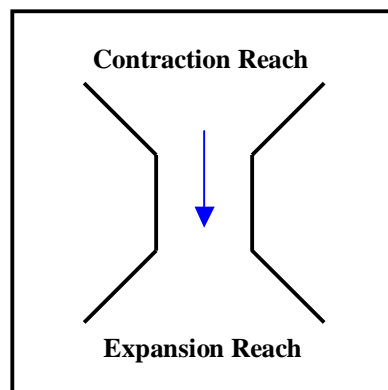

Figure 2.2. Planform View of a Contraction Reach and Expansion Reach

Energy losses due to expansion and contractions along a reach are accounted for through appropriate coefficients. Once an appropriate coefficient is determined, the coefficient is multiplied by the velocity head in order to calculate the energy loss. Equation 2.11 and Equation 2.12 present equations for calculating minor loss due to expansions or contractions, respectively:

$$h_e = C_e \left(\frac{\alpha_2 \overline{v_2}^2}{2g} - \frac{\alpha_1 \overline{v_1}^2}{2g} \right)$$ **Equation 2.11**

where:

α_1 = kinetic energy correction coefficient at the downstream cross section;

α_2 = kinetic energy correction coefficient at the upstream cross section;

C_e = coefficient of expansion;

g = acceleration of gravity (ft/s^2);

h_e = minor loss due to channel expansion at a cross section (ft);

$\overline{v_1}$ = average velocity at the downstream cross section (ft/s); and

$\overline{v_2}$ = average velocity at the upstream cross section (ft/s).

$$h_c = C_c \left(\frac{\alpha_2 \overline{v_2}^2}{2g} - \frac{\alpha_1 \overline{v_1}^2}{2g} \right)$$ **Equation 2.12**

where:

α_1 = kinetic energy correction coefficient at the downstream cross section;

α_2 = kinetic energy correction coefficient at the upstream cross section;

C_c = coefficient of contraction;

g = acceleration of gravity (ft/s^2);

h_c = minor loss due to channel contraction at a cross section (ft);

$\overline{v}_1$ = average velocity at the downstream cross section (ft/s); and

$\overline{v}_2$ = average velocity at the upstream cross section (ft/s).

Typical values for the coefficients of expansion and contraction in a subcritical flow regime are given in Table 2.1, which was published by the USACE in the *HEC-RAS River Analysis System Users Manual Version 3.0* (USACE, 2001b).

Table 2.1. Contraction and Expansion Coefficients (USACE, 2001b)

Subcritical Flow Contraction and Expansion Coefficients	Contraction	Expansion
No Transition Loss Computed	0.00	0.00
Gradual Transitions	0.10	0.30
Typical Bridge Sections	0.30	0.50
Abrupt Transitions	0.60	0.80

2.2.5 Froude Number

In 1-D, steady-state, gradually-varied flow analysis, it is important to note the effect of gravity on the state of the flow. Effect of gravity on the state of flow is represented by a ratio of inertial forces to gravitational forces (Chow, 1959). The ratio of inertial forces to gravitational forces has been termed Froude number and is presented in Equation 2.13:

$$Fr = \frac{\overline{v}}{\sqrt{gH_D}}$$
                                                                    Equation 2.13

where:

Fr = Froude number;

g = acceleration of gravity (ft/s^2);

H_D = hydraulic depth (ft); and

$\overline{v}$ = average velocity at a cross section (ft/s).

Hydraulic depth is defined in Equation 2.14:

$$H_D = \frac{A}{w}$$

Equation 2.14

where:

A = cross-sectional area normal to the direction of flow (ft^2);

H_D = hydraulic depth (ft); and

w = top width of a cross section along the water surface (ft).

For rectangular cross sections, hydraulic depth is assumed equal to flow depth. When the Froude number is equal to one, the flow is termed critical flow. Critical flow is the condition where elementary waves can no longer propagate upstream (Bitner, 2003). If the Froude number is greater than one, the flow is termed supercritical flow. Supercritical flow is characterized by high velocities where inertial forces become dominant at a cross section. If the Froude number is less than one, then the flow is termed subcritical flow. Subcritical flow is characterized by low velocities and is dominated by gravitational forces (Chow, 1959).

2.3 STANDARD STEP METHOD

Based on the concept of conservation of energy, the standard step method uses fundamental hydraulic equations to iteratively calculate water-surface profiles and energy grade lines. Conservation of energy states that "within some problem domain, the amount of energy remains constant and energy is neither created nor destroyed. Energy can be converted from one form to another but the total energy within the domain remains fixed" (Benson, 2004). Iteratively, the standard step method applies conservation of energy using the energy equation to calculate water-surface elevations

and energy grade lines along the reach. For the purpose of the standard step, the energy

equation is written as:

$$y_2 + z_2 + \frac{\alpha_2 \bar{v}_2{}^2}{2g} = y_1 + z_1 + \frac{\alpha_1 \bar{v}_1{}^2}{2g} + h_t \qquad \textbf{Equation 2.15}$$

where:

α_1 = kinetic energy coefficient at the downstream cross section;

α_2 = kinetic energy coefficient at the upstream cross section;

g = acceleration of gravity (ft/s^2);

h_t = total energy loss between adjacent cross sections (ft);

$\bar{v}_1$ = average velocity at the downstream cross section (ft/s);

$\bar{v}_2$ = average velocity at the upstream cross section (ft/s);

y_1 = flow depth at the downstream cross section (ft);

y_2 = flow depth at the upstream cross section (ft);

z_1 = bed elevation at the downstream cross section (ft); and

z_2 = bed elevation at the upstream cross section (ft);

Total energy loss is equal to Equation 2.16 between adjacent cross sections:

$$h_t = h_f + h_e + h_c \qquad \textbf{Equation 2.16}$$

where:

h_c = minor loss due to channel contraction (ft);

h_e = minor loss due to channel expansion (ft);

h_f = energy loss due to friction (ft); and

h_t = total energy loss between adjacent cross sections (ft).

Figure 2.3 illustrates the backwater computation between adjacent cross sections using the energy equation where Q denotes discharge, EGL denotes energy grade line, and XS denotes cross section.
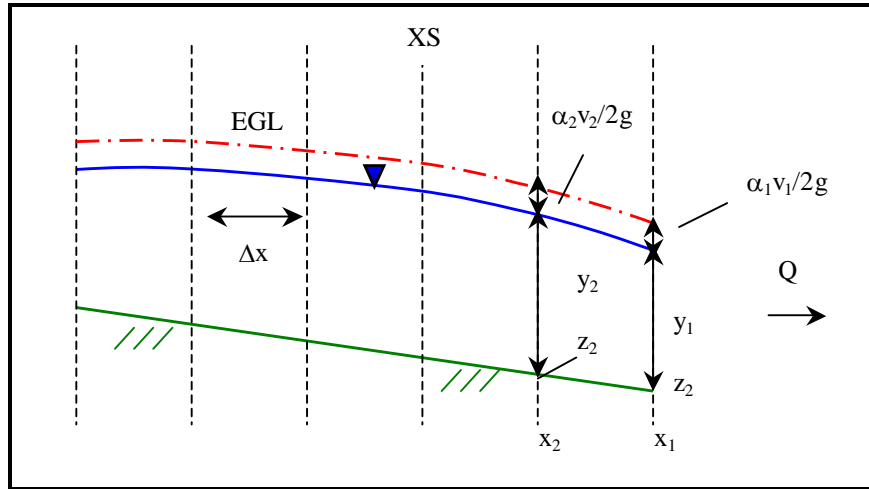

Figure 2.3. Standard Step Method

2.3.1 Standard Step Method Algorithm

The standard step method is one of the coded algorithms in HEC-RAS. If the flow is subcritical, HEC-RAS iteratively calculates a water-surface profile and energy grade line beginning with the most downstream cross section. If the flow is supercritical, HEC-RAS calculates a water-surface profile and energy grade line beginning with the most upstream cross section. An outline of the standard step method used in HEC-RAS is obtained from the *HEC-RAS River Analysis System Hydraulic Reference Manual* and is stated below (USACE, 2001a):

1. Assume a water-surface elevation at an upstream cross section (or downstream cross section if a supercritical profile is being calculated).

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2. Based on the assumed water-surface elevation, determine the corresponding K and v.

3. With values from Step 2, compute $\overline{S}_f$ and solve Equation 2.16 for h_t. $\overline{S}_f$ is calculated using the average conveyance method, the default method in HEC-RAS.

4. With values from Step 2 and Step 3, solve Equation 2.15 for water-surface elevation at the upstream cross section. The water-surface elevation at the upstream cross section is obtained by rearranging Equation 2.15 to Equation 2.17:

$$WSE_2 = y_2 + z_2 = y_1 + z_1 + \left(\frac{\alpha_1 \overline{v_1}^2}{2g} - \frac{\alpha_2 \overline{v_2}^2}{2g} \right) + h_t \qquad \textbf{Equation 2.17}$$

where:

α_1	=	kinetic energy coefficient at downstream cross section;
α_2	=	kinetic energy coefficient at upstream cross section;
g	=	acceleration of gravity (ft/s^2);
h_t	=	total energy loss between adjacent cross sections (ft);
$\overline{v}_1$	=	average velocity at downstream cross section (ft/s);
$\overline{v}_2$	=	average velocity at upstream cross section (ft/s);
WSE_2	=	water-surface elevation at the upstream cross section (ft);
y_1	=	flow depth at downstream cross section (ft);
y_2	=	flow depth at upstream cross section (ft);
z_1	=	bed elevation at downstream cross section (ft); and
z_2	=	bed elevation at upstream cross section (ft).

5. Compare the computed value of the water-surface elevation at the upstream cross section with the value assumed in Step 1, repeat Step 1 through Step 5 until the values agree to within 0.01 ft, or a user-defined tolerance.

In order to start the iterative procedure, a known boundary condition is entered by the user. A boundary condition must be established at the most downstream cross section for a subcritical flow profile and at the most upstream cross section for a supercritical flow profile. Four options are presented in HEC-RAS to establish one boundary condition. The four boundary condition options include the following:

1. known water-surface elevation;

2. critical depth;

3. normal depth; and

4. rating curve.

Critical depth is defined as the flow depth when $Fr = 1$. Normal depth is defined as the depth corresponding to uniform flow (Chow, 1959). Normal depth is calculated after the user enters the bed slope downstream of the study reach. The bed slope is equal to the energy slope for normal depth and, therefore, used in the flow resistance equation to calculate normal depth (USACE, 2001a).

2.4 HEC-RAS FORMAT

A brief discussion is needed to define terminology in HEC-RAS for a steady-state, gradually-varied flow analysis. In this analysis, HEC-RAS Version 3.1.2 was used. A project refers to the HEC-RAS model and encompasses ns, geometry data files, and steady flow files for a particular river system (USACE, 2001b). A project is broken

down into various plans. Each plan represents a "specific set of geometric data and flow data" (USACE, 2001a). Channel geometry data such as survey information, channel lengths, Manning's *n*-values, contraction coefficients, and expansion coefficients are entered into a geometry file. Discharges and boundary conditions are entered into a steady flow file. Once the appropriate information is entered in the geometry file and steady flow file, the defined plan is run in a steady flow analysis. A diagram illustrating the HEC-RAS outline is shown in Figure 2.4.
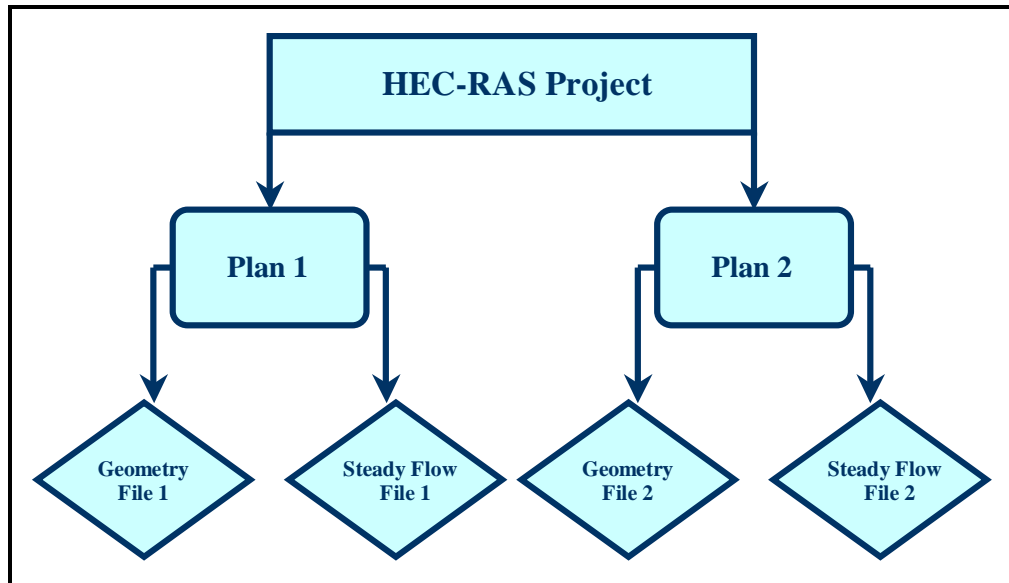

Figure 2.4. HEC-RAS Format

2.5 PREVIOUS STUDIES ON CALCULATING WATER-SURFACE ELEVATIONS IN MEANDER BENDS WITH BENDWAY WEIRS

Previous studies have been completed that used HEC-RAS to calculate water-surface elevations in meander bends incorporating bendway weirs. One study was completed by Breck (2000) at Montana State University. Breck used HEC-RAS Version

2.2 for the purpose of modeling water-surface profiles over **a single bendway weir**. This study was completed for the Highwood Creek watershed, which is located in Central Montana, east of Great Falls. Figure 2.5 locates Highwood Creek in the vicinity of the project site. As Figure 2.5 illustrates, the valley gradient is relatively flat in the vicinity of the project site and sediment deposits tend to be coarse. Flat valley gradient and coarse sediment deposits fill existing channels and force the stream to move laterally. In order to restrict the channel from lateral movement, stream restoration, and bank-stabilization techniques were initiated in the spring of 1996.
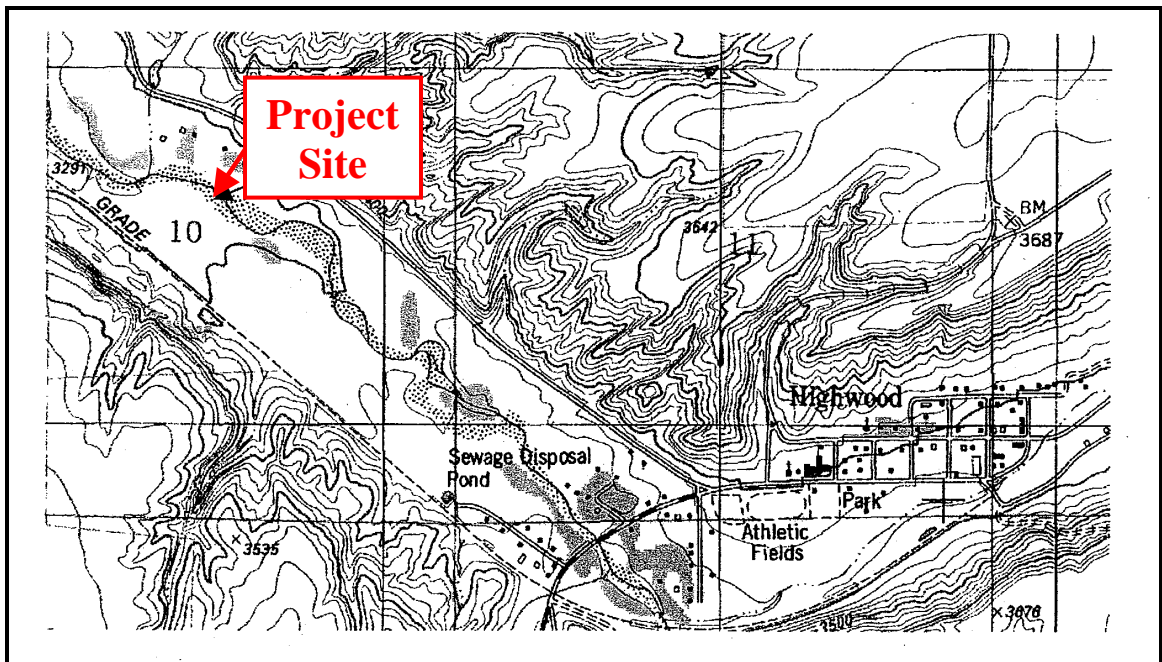

Figure 2.5. Highland Park Map (adapted from Breck (2000))

The project reach was fairly prismatic, approximately 200 ft in length. Five bendway weirs and a vortex weir were constructed along the reach. A vortex weir is a U- or V-shaped, instream rock structure typically composed of native material (Rosgen, 1996).

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In order to build a HEC-RAS model for Highwood Creek, the following data were collected in the field:

1. Flow rate measurements using two methods:

   a. current meter; and

   b. United States Geological Survey (USGS) Database.

2. Manning's *n*-values:

   a. derived from roughness coefficient tables outlined in *Open-Channel Hydraulics* (Chow, 1959).

3. Topographical survey using a total station surveying device which surveyed:

   a. cross section upstream of reach;

   b. cross section downstream of reach; and

   c. water-surface elevations at upstream and downstream cross section.

In addition to the surveyed cross sections upstream and downstream of the study reach, survey data needed to be collected at the bendway weir. Two methods were presented by Breck to survey the bendway weir. Method 1 established five cross sections spaced equally, starting upstream and ending downstream of the bendway weir. Figure 2.6 illustrates the marked cross sections (XS) along the study reach. XS2 through XS6 illustrate the cross-section spacing across the bendway weir. Water-surface elevations were also collected at these cross sections. Unlike Method 1, Method 2 used "one cross section starting at the downstream end of the weir, perpendicular to the study reach, with points being taken along the main body of the structure and continuing perpendicular to the channel at the upstream end." Cross sections were also surveyed upstream and downstream of the bendway weir. Method 2 was used for the ease of collecting data but

the method was not used in the analysis since the survey did not provide enough detail to accurately calculate water-surface elevations across a bendway weir.
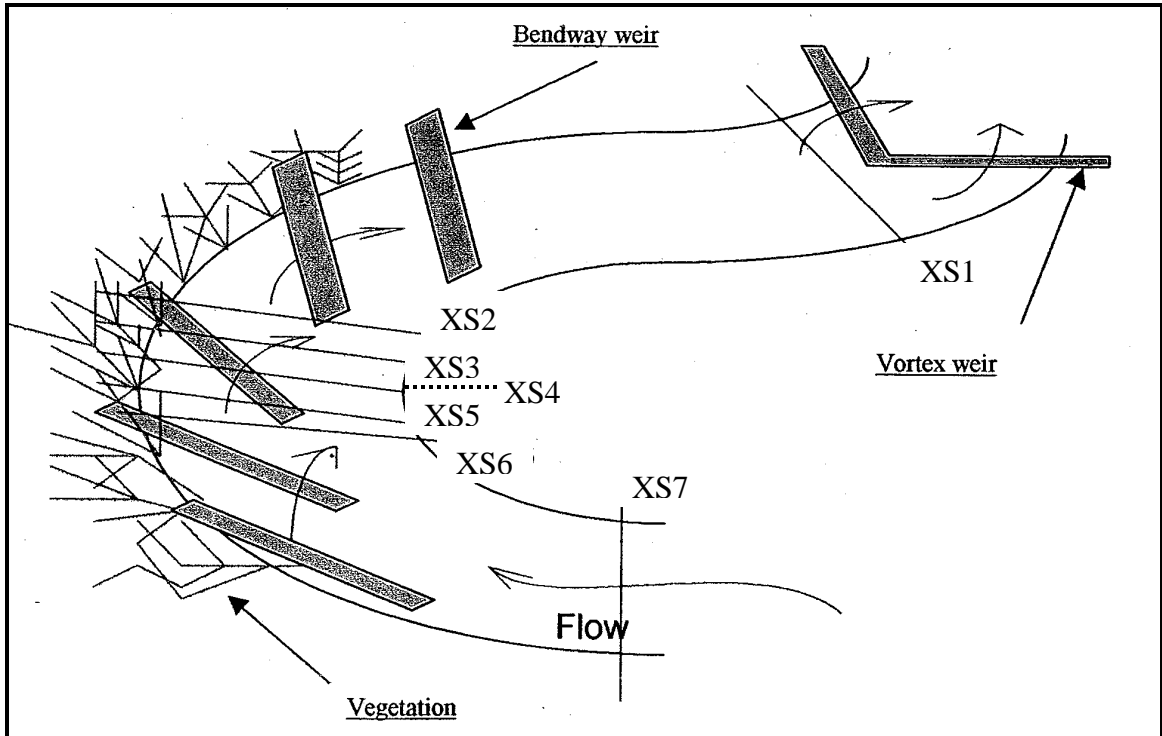

Figure 2.6. Study Reach Survey (adapted from Breck (2000))

From the field data, multiple HEC-RAS models were built in order to determine what methodology produced the most accurate output of water-surface elevations. Seven models, defined as "Options," were built in HEC-RAS and each model is outlined in Table 2.2. Each option assumed Manning's *n* was determined from field data and, therefore, further calibration of Manning's *n* was not required as part of the HEC-RAS analysis.

Table 2.2. Model Option Descriptions (Breck, 2000)

Model Option	Description
1	Survey Method 1 with interpolated cross sections between Station 2 and Station 6; ineffective flow lines on the outside of bendway weir.
2	Survey Method 1 with interpolated cross sections between Station 2 and Station 6; blocked obstructions replace bendway-weir profile in cross-section survey.
3	Survey Method 1 without additional options.
4	Survey Method 2 without additional options.
5	Survey Method 2 with one ineffective flow area.
6	Survey Method 2 with one blocked obstruction.
7	Partial blocked obstruction with ineffective flow areas.

Results of water-surface elevations and flow depths calculated by HEC-RAS confirmed that Option 1 and Option 2 were the most accurate HEC-RAS models. Breck summarized the accuracy of Option 1 and Option 2, and these results are shown in Table 2.3. From these results, Breck noted that the difference between Option 1 and Option 2 is not significant, but by adding additional flow rates over various weir dimensions might determine the superior option. Breck (2000) also noted that Option 1 and Option 2 might show more accurate water-surface elevations if further calibration of Manning's *n* was added to the scope of the analysis.

Table 2.3. Option 1 and Option 2 Accuracy (Breck, 2000)

Option	Option 1 Model Flow Depth (ft)	Option 2 Model Flow Depth (ft)	Observed Depth (ft)	Absolute Error (ft)	Absolute Error (ft)
1	0.95	1.09	1.05	0.100	0.040
2	1.01	1.08	1.04	0.030	0.040
1	0.81	0.81	0.92	0.110	0.110
2	0.86	0.86	0.95	0.090	0.090
1	0.87	0.86	0.98	0.110	0.120
2	0.92	0.91	1.00	0.080	0.090
1	0.93	0.92	1.06	0.130	0.140
2	0.98	0.97	1.08	0.100	0.110
1	0.88	0.88	0.92	0.040	0.040
2	0.94	0.94	0.95	0.010	0.010
			Average Absolute Error	**0.080**	**0.079**

2.6 NATURE OF FLOW IN MEANDER BENDS

Unlike straight channels where streamlines are uniform and parallel, meander bends create streamlines that are curvilinear and interwoven. Curvilinear and interwoven streamlines result in spiral currents and secondary currents (Chow, 1959). Spiral currents refer to movement of water particles along a helical path in the general direction of flow (Chow, 1959). In general, when water moves downstream, a channel curve to the right causes a counterclockwise spiral while a channel curve to the left causes a clockwise spiral. Secondary currents refer to velocity components parallel to the cross section. Spiral currents and secondary currents created in a meander bend are the result of the three factors stated by Chow (1959) in *Open-Channel Hydraulics*:

1. friction on the channel walls;

2. centrifugal force; and

3. vertical velocity distribution which exists in the approach channel.

Centrifugal forces cause the phenomenon in meander bends known as superelevation. Superelevation is the difference in water-surface elevation between the outside bank and inside bank along a cross section. Figure 2.7 illustrates superelevation along with the pressure distribution in a meander bend cross section, which creates spiral currents and secondary currents. Development of spiral currents and secondary currents is an additional source of minor losses due to meander bends.
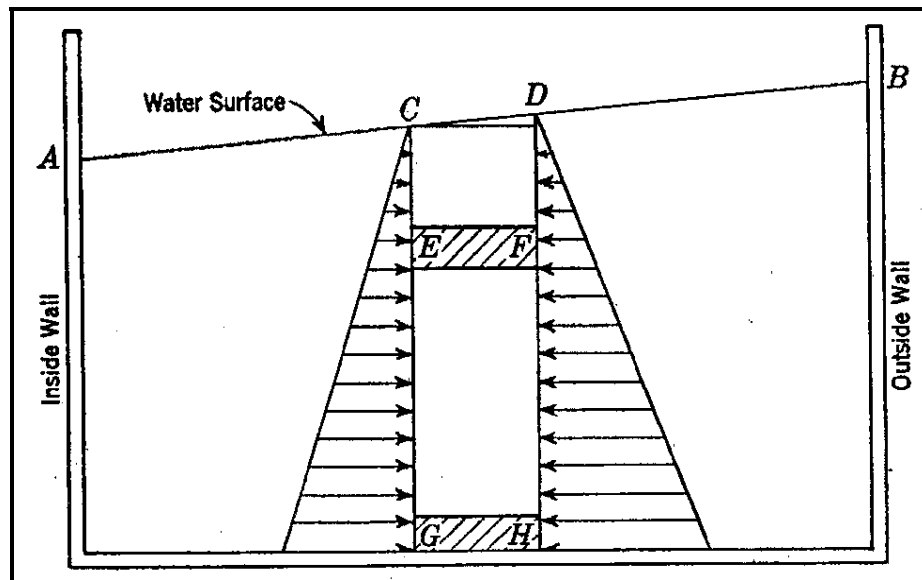

Figure 2.7. Pressure Distribution in a Meander Bend (Mockmore, 1944)

2.7 PREVIOUS STUDIES ON CALCULATING MINOR LOSSES DUE TO MEANDER BENDS

Various studies have been completed to estimate minor loss due to meander bends. Six methods to calculate minor loss due to meander bends are introduced in this section.

2.7.1 Yarnell and Woodward Method

Yarnell and Woodward (1936) stated in the bulletin, *Flow of Water Around 180-Degree Bends*, that minor losses due to bends could be calculated by Equation 2.18:

$$h_{BEND} = C * \frac{w}{r} * \frac{\bar{v}^2}{2g} \qquad \textbf{Equation 2.18}$$

where:

C	=	coefficient of loss;
g	=	acceleration of gravity (ft/s^2);
h_{BEND}	=	energy loss due to bend (ft);
r	=	inner radius (ft);
$\bar{v}$	=	average velocity at a cross section (ft/s); and
w	=	width of channel (ft).

Assuming the channel is rectangular, Table 2.4 contains the list of channel dimensions and representative *C*-values. Yarnell and Woodward point out that coefficients shown in Table 2.4 only apply to the channel dimensions and bend radii stated for the coefficient.

Table 2.4. Yarnell and Woodward *C*-values

| C | Channel Dimensions | | Inner Radius (in.) |
	Length (in.)	Width (in.)	
0.18	10	10	
0.23	5	10	5
0.23	5	10	10

2.7.2 Scobey Method

Chow (1959) reported in his book, *Open-Channel Hydraulics*, a method to calculate minor loss due to meander bends by Scobey in 1933. Scobey stated that minor losses in bends are taken into account by increasing *n*-values by 0.001 for each 20 degree of curvature in 100 ft of channel, but it is uncertain that *n* increases more than 0.002 to 0.003. Scobey's method was developed on the basis of flume tests.

2.7.3 Shukry Method

Chow (1959) reported in his book, *Open-Channel Hydraulics*, a method to calculate minor loss due to meander bends by Shukry in 1950. Shukry used a rectangular, steel flume to demonstrate that minor losses due to flow resistance in bends can be expressed as a coefficient multiplied by the velocity head at a cross section. Equation 2.19 illustrates this expression:

$$h_b = f_c * \frac{\overline{v}^2}{2g}$$

Equation 2.19

where:

f_c = coefficient of curve resistance;

g = acceleration of gravity (ft/s^2);

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h_b = minor loss due to the bend (ft); and

$\bar{v}$ = average velocity at a cross section (ft/s).

Shukry identified four significant parameters in order to classify flow in a bend. These parameters are shown in the following list:

1. r_c/b

2. y/b

3. $\theta/180$

4. Re

where:

b = channel width (ft);

r_c = radius of curvature (ft);

Re = Reynold's number;

θ = deviation angle of the curve; and

y = flow depth (ft).

Reynold's number is expressed by the following equation:

$$Re = \frac{\bar{v}R}{\upsilon}$$ **Equation 2.20**

where:

R = hydraulic radius (ft);

Re = Reynold's number;

υ = kinematic viscosity of a fluid (ft^2/s); and

$\bar{v}$ = average velocity at a cross section (ft/s).

Reynold's number ranged from 10,000 to 80,000 in Shukry's experiments.

2.7.4 Yen and Howe Method

Brater and King (1976) reported in their book, *Handbook of Hydraulics*, a method to calculate minor loss due to meander bends by Yen and Howe in 1942. Yen and Howe reported that minor loss due to meander bends is calculated by multiplying a coefficient by the velocity head at a cross section. Equation 2.21 presents the formula to calculate minor loss due to meander bend:

$$h_b = K_b * \frac{\overline{v}^2}{2g}$$
Equation 2.21

where:

g = acceleration of gravity (ft/s^2);

h_b = minor loss due to bend (ft);

K_b = coefficient of curve resistance; and

$\overline{v}$ = average velocity at a cross section (ft/s).

K_b is equal to 0.38 for a 90° bend having a channel width of 11 in. and a radius of curvature of 5 ft.

2.7.5 Tilp and Scrivner Method

Brater and King (1976) reported in their book, *Handbook of Hydraulics*, a method to calculate minor loss due to meander bends by Tilp and Scrivner in 1964. Tilp and Scrivner suggested that minor losses due to bends could be estimated from the following equation:

$$h_b = 0.001 * \left(\Sigma \Delta^\circ\right) * \frac{\overline{v}^2}{2g}$$
Equation 2.22

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where:

g = acceleration of gravity (ft/s^2);

h_b = minor loss due to bend (ft);

$\Sigma\Delta°$ = summation of deflection angles; and

$\bar{v}$ = average velocity at a cross section (ft/s).

Tilp and Scrivner developed this equation based on large, concrete-lined canals.

2.7.6 Lansford Method

Robertson introduced an equation by Lansford in the *American Society of Civil Engineers Paper No. 2217* (Mockmore, 1944). Lansford reported that difference in pressure of fluid flowing in a bend could be expressed by Equation 2.23:

$$\Delta h = \frac{2b}{r_c} * \frac{\bar{v}^2}{2g}$$

Equation 2.23

where:

b = channel width (ft);

Δh = difference in pressure of fluid flowing (ft);

g = acceleration of gravity (ft/s^2);

r_c = radius of curvature (ft); and

$\bar{v}$ = average velocity at a cross section (ft/s).

This relationship is due to centrifugal forces of water acting on a channel bend and was developed for closed conduit bends.

2.7.7 Summary

Limitations were required to calculate minor loss due to meander bend in each method stated in Section 2.7. For instance, the Yarnell and Woodward method noted that the coefficient of loss required to calculate energy loss due to the bend in Equation 2.18 only applied to design flumes with dimensions specified in Table 2.4. Table 2.4 indicated that the maximum channel length and channel width was 10 in. The Shukry method used Equation 2.19 to calculate minor loss due to the bend using a coefficient of curve resistance. Coefficient of curve resistance was developed for a rectangular, steel flume with Reynold's numbers ranging from 10,000 to 80,000. The Yen and Howe method noted that minor loss due to the bend is calculated in Equation 2.21 using a coefficient of curve resistance. Coefficient of curve resistance is limited to a design flume with a 90° bend, 11-in. channel width, and 5-ft radius of curvature.

Constraints required to calculated the minor loss due to meander bend limited the applicability of each method. A method needs to be developed in order to calculate minor loss due to meander bend in open-channel systems for an array of bend angles, channel widths, and channel lengths.