Some background information:

The final aim of large scale landslide hazard analysis (scales larger than 1:10,000) is to create quantitative hazard maps. The hazard degree can be expressed by the Safety Factor, which is the ratio between the forces that make the slope fail and those that prevent the slope from failing. F-values larger than 1 indicate stable conditions, and F-values smaller than 1 unstable. At F=1 the slope is at the point of failure.

\[ F = \frac{cA + W\cos\beta \tan\phi}{W\sin\beta} \]

in which:
- \( c \) = cohesion (Pa = N/m²).
- \( A \) = length of the block (m).
- \( W \) = weight of the block (kg).
- \( \beta \) = slope surface inclination (°).
- \( \phi \) = angle of shearing resistance (°)

According to the formula:
- F < 1 unstable slope conditions,
- F = 1 slope is at the point of failure,
- F > 1 stable slope conditions
Some background information:
Many different models exist for the calculation of Safety Factors. Here we will use one of the simplest models, the so-called *infinite slope model*. This one dimensional model describes the slope stability of slopes with an infinitely large failure plane. It can be used in a GIS, as the calculation can be done on a pixel basis. The pixels in the parameter maps can be considered as homogeneous units. The effect of the neighboring pixels is not considered, and the model can be used to calculate the stability of each individual pixel, resulting in a hazard map of safety factors. The safety factor is calculated according the following formula (Brunsden and Prior, 1979):

\[
F = \frac{c' + (\gamma - m \gamma_w) z \cos^2 \beta \tan \phi'}{\gamma z \sin \beta \cos \beta}
\]

in which:

- \(c'\) = effective cohesion (Pa= N/m²).
- \(\gamma\) = unit weight of soil (N/m³).
- \(m\) = \(z_w/z\) (dimensionless).
- \(\gamma_w\) = unit weight of water (N/m³).
- \(z\) = depth of failure surface below the surface (m).
- \(z_w\) = height of watertable above failure surface (m).
- \(\beta\) = slope surface inclination (°).
- \(\phi'\) = effective angle of shearing resistance (°).

The infinite slope model can be used on profiles as well as on pixels. The entire analysis requires first the preparation of the data base. The parts on groundwater modelling and the modelling of seismic acceleration are not shown here. For more information see Van Westen (1993). In this exercise only the calculation of average safety factors will be done for different scenarios. These average safety factor maps could be used in the creation of failure probability maps.

Visualization of the input data

In this exercise the slope stability analysis is made by using only two parameter maps: *Soildepth* (thickness of soil) and *Slope_map* (slope angles in degrees).

- Open the maps *Soildepth* and *Slope_map* and check the values in the maps. Click OK in the Display Options dialog box. The map is displayed.
We assume that:
the depth of the possible failure plane is taken at the contact of
the soil and the underlying weathered rock material.
All soils have the same values for Cohesion, friction angle and
unit weight.

\[ M : \]
\[ \frac{Z_w}{Z} = \frac{\text{Depth to}}{\text{groundwater /}} \frac{\text{Depth to failure}}{\text{surface}} \]

In the first part of the exercise we will calculate the stability of the soil cover using only one single value of cohesion, friction angle and bulk density. The consequence of this is that safety factors will not be calculated for the entire area, but only for the areas where there is soil overlying the bed rock.

Besides soil depth which is assumed to be the same as the depth to the failure surface, and the slope of the terrain, we also need to know the other parameters of the infinite slope formula. From laboratory analysis the following average values are known:

- \( c' = \) effective cohesion (Pa= N/m\(^2\)) = 11000 Pa
- \( \gamma_w = \) unit weight of water (N/m\(^3\)) = 10000 N/m\(^3\)
- \( z = \) depth of failure surface below the surface (m) = map
- \( \beta = \) slope surface inclination (°) = map
- \( \phi' = \) effective angle of shearing resistance (°) = 32°
- \( \tan(\phi') = \) tangent of the effective angle of shearing resistance = 0.625

The only unknown parameter yet is the depth of the water table. In the formula this is expressed as the value \( m \), which is the relation between the depth of the water table and the depth of the failure surface.

**Preparation of the data**

Before you can start with the analysis, you need to reorganize the map Slope_map. In the calculation we need three parameters that are derived from the slope:

- \( \sin(\text{slope}) = \) the sine of the slope
- \( \cos(\text{slope}) = \) the cosine of the slope
- \( \cos^2(\text{slope}) = \cos(\text{slope})^2 \cos(\text{slope}) \)

The ILWIS functions for sine and cosine only work with input values in radians, while our map Slope is in degrees. Therefore we need to convert to slope map from degrees to radians first.

\[ \text{Degrad}(\text{Slope}) \text{ degrees to radians function: slope*2\pi/360} \]

- Type the following formula on the command line:
  \[ \text{Slrad} := \text{degard}(\text{Slope_map}) \]
  Accept the default minimum, maximum and precision.
- Open the result map and compare the values of the map Slrad with those of the map Slope_map. With the pixel information click in some points of the map and read the values in degree and in radians. For example: you can type in the command line of ILWIS the following formula:
  \[ ?10.77*2*3.14/360 \]

Now you have the slope in radians, and you can calculate the sine and cosine. You will calculate individual maps for these so that the Safety factor formula (formula 6.1) can be calculated easier.
Besides many internal pre-programmed functions, ILWIS gives the user an opportunity to create new functions. Especially when you need to execute certain calculations which require a lot of typing work several times, user-defined functions may be time saving. A user-defined function is an expression which may contain any combination of operators, functions, maps and table columns.

Functions

Now all necessary parameters for the formula are known, except for the parameter $m$ related to the groundwater depth.

Creating a function for the infinite slope formula

In the following sections you will use the infinite slope formula extensively for different scenarios, and different input data. To avoid that you have to retype the formula each time, it is better to create a user-defined function for it.

In this dialog box you can edit the expression of the function. Now the expression is:

$$
\begin{align*}
\text{Fs} &= \frac{(\text{Cohesion} + (\text{Gamma} - M*\text{Gammaw}) \times Z \times \text{Co}^2 \times \text{Tanphi})}{\text{Gamma} \times Z \times \text{Si} \times \text{Co}} \\
\text{Description: Safety factor.}
\end{align*}
$$

- Double-click New Function in the operations list. The Create function dialog box is opened.
- Type for the Function Name: Fs
- Type for the expression:
  $$(\text{Cohesion} + (\text{Gamma} - M*\text{Gammaw}) \times Z \times \text{Co}^2 \times \text{Tanphi}) / (\text{Gamma} \times Z \times \text{Si} \times \text{Co})$$
- Click OK. The Edit Function dialog box is opened. Click OK.
Function $fs(Value\ Cohesion, Value\ Gamma, Value\ M, Value\ GammaW, Value\ Z, Value\ Co2, Value\ Tanphi, Value\ Si, Value\ Co) : Value$

Begin
Return $(Cohesion + (\Gamma - M \times GammaW) \times Z \times Co2 \times Tanphi) / (\Gamma \times Z \times Si \times Co)$
End;

As you can see the function contains the following variables (listed in the first line):
- $Value\ Cohesion$: the value for the effective cohesion.
- $Value\ Gamma$: the value for the unit weight of soil.
- $Value\ m$: the value for the relation $zw/w$.
- $Value\ GammaW$: the value for the unit weight of water.
- $Value\ z$: the value for the depth of failure surface below the surface.
- $Value\ Co2$: the value for the square of the cosine of the slope.
- $Value\ Tanphi$: the value for the tangent of the effective angle of shearing resistance.
- $Value\ Si$: the value for the sine of the slope.
- $Value\ Co$: the value for the cosine of the slope.

However, a number of these variables are fixed. You will use them for all calculations: $Value\ Cohesion$ (10000 Pa), $Value\ GammaW$ (10000 N/m$^3$), $Value\ Z$ (raster map $Soildepth$), $Value\ Co2$ (raster map $Co2$), $Value\ Tanphi$ (0.625), $Value\ Si$ (raster map $Si$), and $Value\ Co$ (raster map $Co$).

So you can simplify the function considerably, so that it looks like:

Function $fs(Value\ Gamma, Value\ M) : Value$

Begin
Return $(10000 + ((\Gamma - m \times 10000) \times Soildepth \times Co2 \times 0.625)) / (\Gamma \times Soildepth \times Si \times Co)$
End;

As you can see there are only two variables: $Value\ Gamma$ and $Value\ M$.

- Edit the Function until it is the same as above. Click OK.

**Calculating Safety Factors for groundwater scenarios**

Now that the function is created, you can start to calculate safety factor maps for different scenarios. In the first part you will calculate the safety factors for different scenarios where only rainfall is the triggering factor. You will not yet look at the influence of an earthquake.
Dry condition

You will first calculate the safety factor for the soils under the assumption that the soil is completely dry. In that case the parameter m is equal to zero.

Remember the other parameter that were given on the previous page:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c' )</td>
<td>Effective cohesion (( \text{Pa} = \text{N/m}^2 ))</td>
<td>11000 Pa</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Unit weight of soil (( \text{N/m}^3 ))</td>
<td>11000 N/m^3</td>
</tr>
<tr>
<td>( \gamma_w )</td>
<td>Unit weight of water (( \text{N/m}^3 ))</td>
<td>10000 N/m^3</td>
</tr>
<tr>
<td>( z )</td>
<td>Depth of failure surface below the surface (m)</td>
<td>= map Soildepth</td>
</tr>
<tr>
<td>( m )</td>
<td>Relation zw/z (dimensionless)</td>
<td>0</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Slope surface inclination ((^\circ))</td>
<td>= map Slope_map</td>
</tr>
<tr>
<td>( \phi' )</td>
<td>Effective angle of shearing resistance ((^\circ))</td>
<td>32 (^\circ)</td>
</tr>
<tr>
<td>( \tan(\phi') )</td>
<td>Tangent of the effective angle of shearing resistance</td>
<td>0.625</td>
</tr>
<tr>
<td>( \sin(\beta) )</td>
<td>Sine of slope angle</td>
<td>= map Si</td>
</tr>
<tr>
<td>( \cos(\beta) )</td>
<td>Cosine of slope angle</td>
<td>= map Co</td>
</tr>
<tr>
<td>( \cos^2(\beta) )</td>
<td>Square of the cosine of slope angle</td>
<td>= map Co^2</td>
</tr>
</tbody>
</table>

Now you can start with the actual calculation of the average safety factor map representing the situation under dry conditions. The two variables for the function \( fs \) are 11000 (Value Gamma) and 0 (Value M).

- Type the following formula on the command line:
  
  \[ F_{dry} := fs(11000, 0) \]

- Make sure the georeference Somewhere is used. Use a minimum of 0, a maximum of 100, and a precision of 0.1.

- Open the result map and compare the values of the map \( F_{dry} \) with those of the input maps. Calculate the safety factor manually for some pixels with the Pocket line calculator or the Windows calculator, using the infinite slope formula

The resultant map \( F_{dry} \) will have some pixels with missing values indicated by a question mark (?). You can investigate these pixels and see that the values of the pixels cannot be calculated either because they lack soil or because they are flat areas. Both conditions indicate stability and thus can be safely grouped as stable.

As you can imagine a situation with a completely dry situation does not occur in a tropical region such as RiskCity, which receives quite a lot of rainfall each year. In any case the map \( F_{dry} \) gives the most stable situation. Let us see how much percent of the area is unstable under these conditions. In order to know that we will first classify the map \( F_{dry} \) into three classes:

- Unstable = safety factor lower than 1
- Critical = safety factor between 1 and 1.5
- Stable = safety factor above 1.5

- Create a new domain Stabil (type class, group) with the following three classes:
  
<table>
<thead>
<tr>
<th>Boundary Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Unstable</td>
</tr>
<tr>
<td>1.5</td>
<td>Critical</td>
</tr>
</tbody>
</table>
RiskCity exercise: Deterministic landslide hazard assessment

100 Stable

• Use the Slicing operation to classify the map Fdry with the domain Stabil into the map Fdryc.

• Calculate a histogram of the map Fdryc and write down the percentages of the three classes in a table on a sheet of paper with the column name Dry. Later we will calculate the values for other situations.

The percentage of the pixels classified as unstable gives you an indication of the error, since the occurrence of unstable pixels under fully dry conditions is not possible.

Completely saturated condition

The next scenario that you will evaluate is a condition in which the slopes are completely saturated. This is also not a very realistic situation, but it will give us the most pessimistic estimation of slope stability, with only one triggering factor involved (rainfall leading to high perched watertables).

When we have a saturated soil, the m factor from the infinite slope formula is equal to 1. This means that the watertable is at the surface. There is also another parameter that will vary when the soil is completely saturated, which is γ:

\[
\begin{align*}
    c' &= \text{Effective cohesion (Pa}\,=\,\text{N/m}^2) = 11000 \text{ Pa} \\
    \gamma &= \text{Unit weight of soil (N/m}^3) = 16000 \text{ N/m}^3 \\
    \gamma_w &= \text{Unit weight of water (N/m}^3) = 10000 \text{ N/m}^3 \\
    z &= \text{Depth of failure surface below the surface (m)} = \text{map Soildepth} \\
    m &= \text{Relation } z_w/z \text{ (dimensionless)} = 1 \\
    \beta &= \text{Slope surface inclination (°)} = \text{map Slope_map} \\
    \phi' &= \text{Effective angle of shearing resistance (°)} = 32° \\
    \tan(\phi') &= \text{Tangent of the effective angle of shearing resistance} = 0.625 \\
    \sin(\beta) &= \text{Sine of slope angle} = \text{map Si} \\
    \cos(\beta) &= \text{Cosine of slope angle} = \text{map Co} \\
    \cos^2(\beta) &= \text{Square of the cosine of slope angle} = \text{map Co2}
\end{align*}
\]

The two variables for the function $fs$ are 16000 (value γ) and 1 (value m).

• Type the following formula on the command line:

$$Fsat:=fs(16000,1)\ldots$$

• Use a minimum of 0, a maximum of 100, and a precision of 0.1. Change the GeoReference to "Somewhere".

• Open the result map and compare the values of the map Fsat with the maps Fdry and the input map. Calculate the safety factor manually for some pixels with the ILWIS pocket line calculator or the Windows calculator for some pixels, using the formula given above.

• Use the Slicing operation (under image procession) to classify the map Fsat with the domain Stabil into the map Fsatic.

• Calculate a histogram of the map Fsatic and write down the percentages of the three classes in a table on a sheet of paper with the column name Sat. Compare them with the column Dry.

Now that we have calculated all scenarios, we can compare them. This can be done in a table.

RiskCity exercise 3L2 - 7
• Create a table from the domain **Stabil**.

• Go to Columns, Join and select Table histogram of **Fdryc**; use the column **Npixpct**. Give the name **Dry** to the output column. Accept the default values and click Ok.

• Also join the histogram file of the map **Fsatc**. Give the name **Sat**.

• Click the button Graph in the main menu, Remove the tick mark in the X axis, Select **Dry** as Y axis and click OK. You will see a histogram of the percentage area under different stability classes in dry condition.

• In the Graph window itself, go to **Edit menu, Add Graph** and select and from columns. Select Y axis **Sat**. Now you will see the histograms of the percentage area under different stability classes in dry and saturated conditions side by side.

• Draw conclusions on the effect of the groundwater on the stability of the soils in the area.

**Partially saturated condition**

• Now design some scenarios yourself where you use M values that vary between 0 and 1, and see the effect on the stability.

• Draw conclusions on the effect of the groundwater on the stability of the soils in the area.
For experienced ILWIS users:

**For experienced ILWIS users:**

**Using different values for cohesion and friction angle**
- It is also important to include different values for cohesion, friction angle and unit weight of soils for different soil or lithological types. You can do that by adding columns **Cohesion**, **FrictionAngle** and **Gamma** to the table **Lithology**.

**For experienced ILWIS users:**

**Include earthquake acceleration**
- It is also possible to include earthquake acceleration in the equation for the factor of safety. You could try to make a function for that based on the formula given below, and test this out for a few scenarios of earthquake acceleration and M values.

\[
F = \frac{C' + z(\gamma \cos^2 \beta - \rho a_h N \cos \beta \sin \beta - \gamma_w m \cos^2 \beta \tan \varphi')}{z(\gamma \sin \beta \cos \beta + \rho a_h N \cos^2 \beta)}
\]

- \(C'\) = effective cohesion (Pa)
- \(z\) = depth of failure surface below terrain surface
- \(\gamma\) = unit weight of soil (Nm\(^{-3}\))
- \(\beta\) = terrain surface inclination (degrees)
- \(\rho\) = bulk density (kgm\(^{-3}\))
- \(A_h\) = peak horizontal acceleration in rock (ms\(^{-2}\))
- \(N\) = amplification of seismic acceleration in soil
- \(\gamma_w\) = unit weight of water (Nm\(^{-3}\))
- \(M\) = groundwater/soil thickness ratio \(z_w/z\)
- \(Z_w\) = height of the water table above failure surface (m)
- \(\varphi'\) = effective angle of shearing resistance (degrees)