HECRAS
Basic Principles of Water Surface Profile Computations

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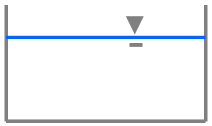
What about water...

- Incompressible fluid
  - must increase or decrease its velocity and depth to adjust to the channel shape
- High tensile strength
  - allows it to be drawn smoothly along while accelerating
- No shear strength
  - does not decelerate smoothly, results in standing waves, good canoeing, air entrainment, etc

What about open channel flow...

- A free surface
- Liquid surface is open to the atmosphere
- Boundary is not fixed by the physical boundaries of a closed conduit



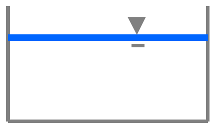
Since the flow is incompressible, the product of the velocity and cross sectional area is a constant. Therefore, the flow must increase or decrease its velocity and depth to adjust to the shape of a channel.

→ **Continuity Equation**

(conservation of mass)

VA = constant
Discharge is expressed as Q = VA

- Q is flow
- V is velocity
- A is cross section area

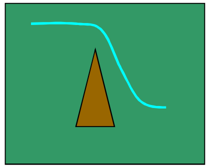


Open Channel Flow - Controls

Definition: A control is any feature of a channel for which a unique depth - discharge relationship occurs.

Weirs/Spillways

Abrupt changes in slope or width



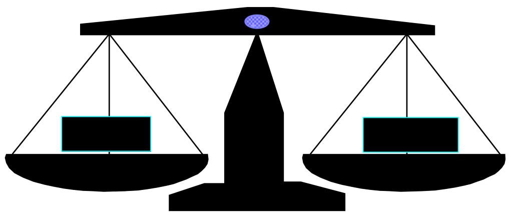
Friction - over a distance

Water goes downhill. What does that mean to us?

- Water flowing in an open channel typically gains energy (kinetic) as it flows from a higher elevation to a lower elevation
- It loses energy with friction and obstructions.

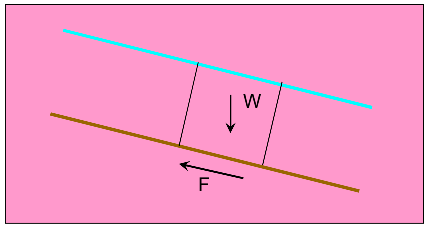
The gravitational forces that are pushing the flow along are in balance with the frictional forces exerted by the wetted perimeter that are retarding the flow.

→ Uniform or Normal Flow



Over a long stretch of a river...

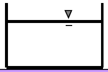
Average velocity: is a function (slope and the resistance to drag along the boundaries)

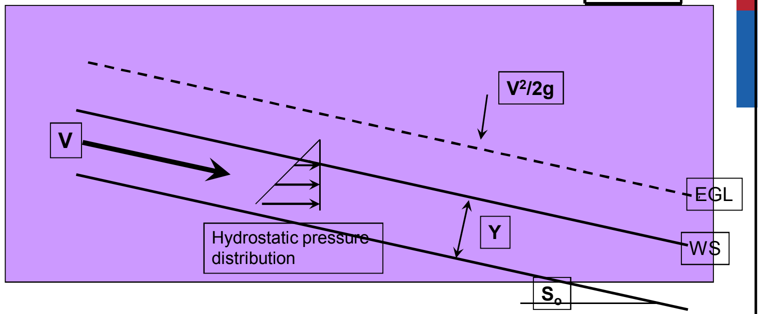


What does that mean?

Assume a Hypothetical Channel

Long prismatic (same section over a long distance) channel
No change in slope, section, discharge





Channel bottom is <u>parallel</u> to water surface is <u>parallel</u> to energy grade line. The streamlines are parallel.

Uniform flow occurs when the gravitational forces are exactly offset by the resistance forces

Uniform flow occurs when:

- Mean velocity is constant from section to section
- Depth of flow is constant from section to section
- Area of flow is constant from section to section

Therefore: It can only occur in very long, straight, prismatic channels where the terminal velocity of the flow is achieved

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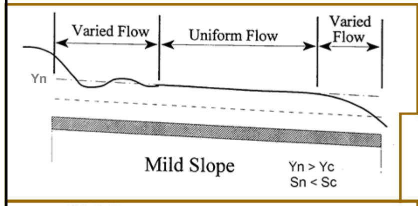
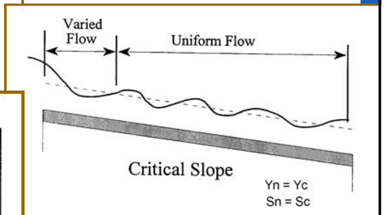
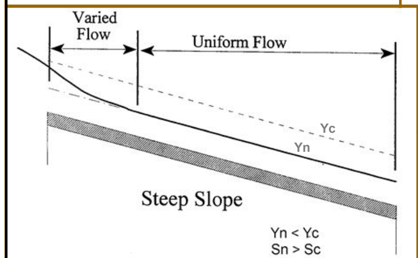
Normal Depth Applies to Different Types of Slopes

'n' stands for normal
's' stands for critical







If the flow is a function of the slope and boundary friction, how can we account for it?

Newton's second law $\sum F = \cancel{ma} \rightarrow 0$

If velocity is constant, then acceleration is zero. So use a simple force balance.

$$\tau_0 PL - W\sin\Theta = 0$$

where

$$\tau_0 = KV^2$$

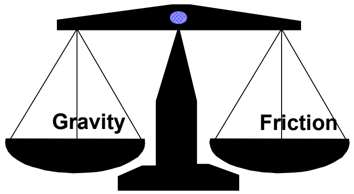
rearrange equation

$$KV^2PL = \gamma AL\sin\Theta$$

$$\Theta small$$

$$\therefore \sin\Theta = S_0$$

$$V = \sqrt{\frac{\gamma}{K}}\sqrt{RS_0}$$



Antoine Chezy (18th century)

$$V = C\sqrt{RS_o}$$

Manning’s equation

$$Q = \frac{1.49}{n}AS^{1/2}R^{2/3}$$

$$Q = flow(cfs)$$

$$n = coeff$$

$$A = area$$

$$S = slope$$

$$P = wetted \quad perimeter$$

$$R = A/P$$

One of the most widely used to account for friction losses

Manning’s Equation - ‘n’ units

$$Q = \frac{1.49}{n}AS^{1/2}R^{2/3}$$

$$R = A/P \qquad L^2/L = L$$

$$L^3/T = (L^2)(L^{2/3})/n$$

L = length (feet, meters, inches, etc)
T = time (seconds, minutes, hours, etc)

Manning’s n

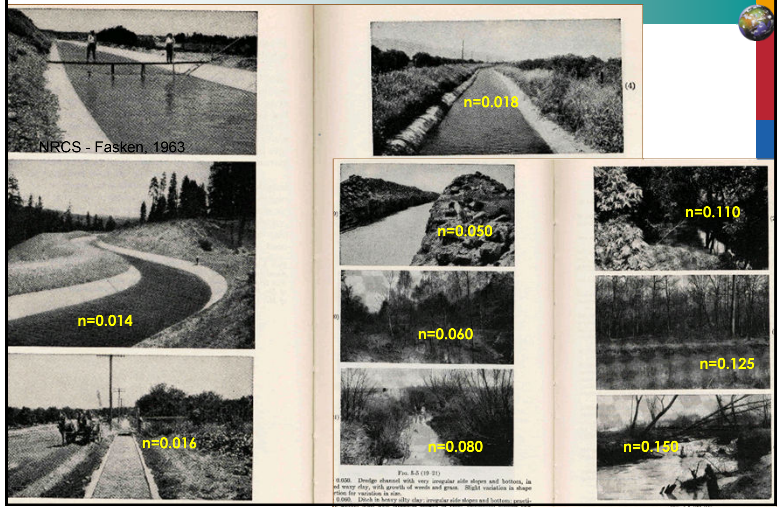
A bulk term, a function of grain size, roughness, irregularities, etc…
Values been suggested since turn of century (King 1918)

Manning's n values for small, natural streams (top width <30m)

Lowland Streams		Minimum	Normal	Maximum
(a)	Clean, straight, no deep pools	0.025	0.030	0.033
(b)	Same as (a), but more stones and weeds	0.030	0.035	0.040
(c)	Clean, winding, some pools and shoals	0.033	0.040	0.045
(d)	Same as (c), but some weeds and stones	0.035	0.045	0.050
(e)	Same as (c), at lower stages, with less effective and sections	0.040	0.048	0.055
(f)	Same as (d) but more stones	0.045	0.050	0.060
(g)	Sluggish reaches, weedy, deep pools	0.050	0.070	0.080
(h)	Very weedy reaches, deep pools and floodways with heavy stand of timber and brush	0.075	0.100	0.150
Mountain streams (no vegetation in channel, banks steep, trees and brush submerged at high stages				
(a)	Streambed consists of gravel, cobbles and few boulders	0.030	0.040	0.050
(b)	Bed is cobbles with large boulders	0.040	0.050	0.070

(Chow, 1959)

Guidance Available: seek bibliography and the WEB



Manning’s n in Steep Channel

- Streams may appear to be super critical but are just fast subcritical
- Jarret’s Eqn (ASCE J. of Hyd Eng, Vol. 110(11)) (R = hydraulic radius in feet)

$$n = 0.39 S^{0.38} R^{-0.16}$$

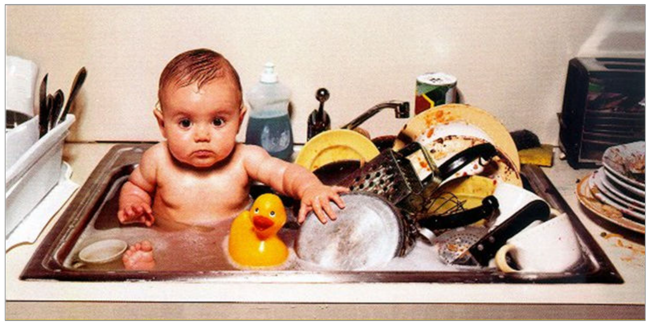
What can we do with Manning’s Equation?

- Can compute many of the parameters that we are interested in:
  - Velocity
  - Trial and error for depth, width, area, etc

$$V = \frac{1.49}{n} S^{1/2} R^{2/3}$$

$$Q = \frac{1.49}{n} A S^{1/2} R^{2/3}$$

So…why make things any more complicated?



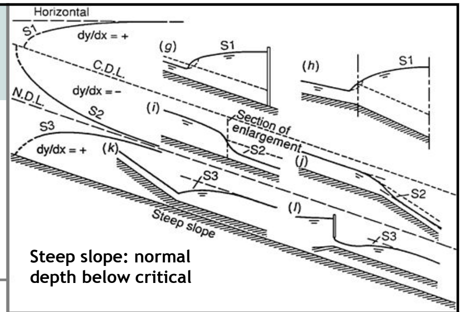
How sensitive is the equation?

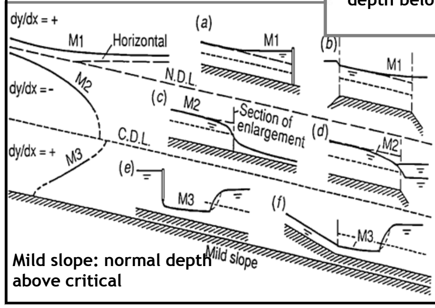
W=100'
d=5'
S=0.004
n=0.035

Q=3700 cfs

n=0.03 to 0.04	→	13% to 17%
d=4.5 to 5.5 ft	→	16% to 17%
w =90 to 110 ft	→	11%
S=0.003 to 0.005	→	12% to 13%
All	→	40% to 70%

How often do we see normal depth in real channels?





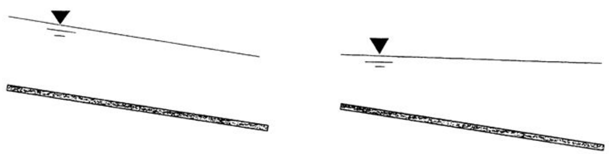
Streams go towards normal depth but seldom get there

Limitations of a Normal Depth Computation:

- Constant Section - natural channel?
- Constant Roughness - overbank flow?
- Constant Slope
- No Obstructions - bridges, weirs, etc

Open Channel Flow is typically varied

B. Uniform vs. Varied



Uniform Flow

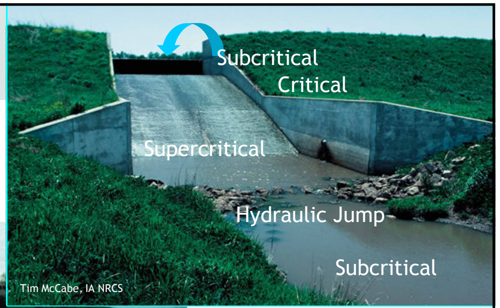
Depth and velocity are constant with distance along the channel.

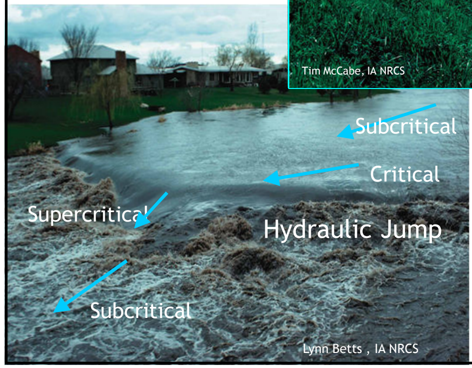
Varied Flow

Depth and velocity vary with distance along the channel.

Figure 2 Uniform versus Varied Flow

Both man made and natural channels





In natural gradually varied flow channels:

Velocity and depth changes from section to section. However, the energy and mass is conserved.

Can use the energy and continuity equations to step from the water surface elevation at one section to the water surface at another section that is a given distance upstream (subcritical) or downstream (supercritical)

HEC-RAS uses the ***one dimensional energy equation*** with energy losses due to friction evaluated with Manning’s equation to compute water surface profiles. This is accomplished with an iterative computational procedure called the Standard Step Method.

How about that energy equation?

- First law of thermodynamics

Bernoulli 's Equation

$$(V^2/2g)_2 + P_2/w + Z_2 = (V^2/2g)_1 + P_1/w + Z_1 + h_e$$

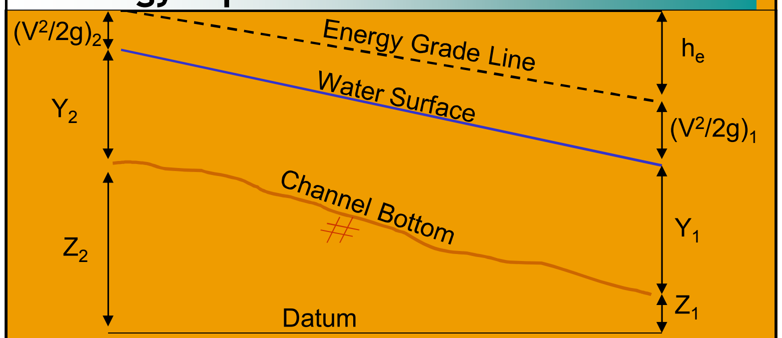
- Kinetic energy + pressure energy + potential energy is conserved

Remember - it's an open channel and a hydrostatic pressure distribution

$$(V^2/2g)_2 + \overset{Y_2}{\cancel{P_2/w}} + Z_2 = (V^2/2g)_1 + \overset{Y_1}{\cancel{P_1/w}} + Z_1 + h_e$$

Pressure head can be represented by water depth, measured vertically (can be a problem if very steep - streamlines converge or diverge rapidly)

Energy Equation

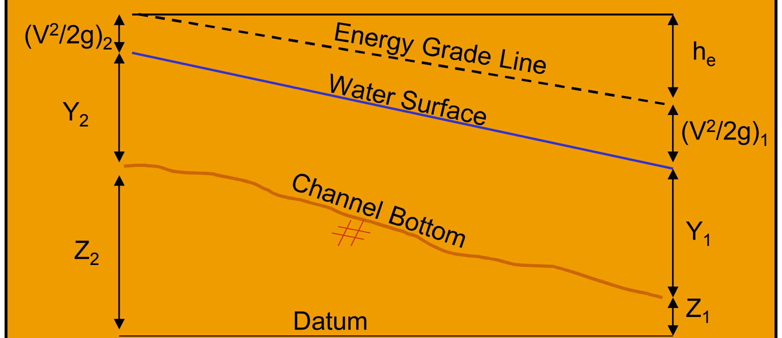


$$(V^2/2g)_2 + Y_2 + Z_2 = (V^2/2g)_1 + Y_1 + Z_1 + h_e$$

Energy Losses

$$h_e = L\bar{S}_f + C\left|\frac{\alpha V_2^2}{2g} - \frac{\alpha V_1^2}{2g}\right|$$

Standard Step Method



$$WS_2 = WS_1 + \frac{1}{2g}(\alpha_1 V_1^2 - \alpha_2 V_2^2) + h_e$$

- Start at a known point.
- How many unknowns?
- Trial and error

HEC-RAS - Computation Procedure

1. Assume water surface elevation at upstream/ downstream cross-section.
2. Based on the assumed water surface elevation, determine the corresponding total conveyance and velocity head
3. With values from step 2, compute and solve equation for 'he'.
4. With values from steps 2 and 3, solve energy equation for WS2.
5. Compare the computed value of WS2 with value assumed in step 1; repeat steps 1 through 5 until the values agree to within 0.01 feet, or the user-defined tolerance.

Energy Loss - important stuff

- Loss coefficients Used:
  - Manning's n values for friction loss
    - very significant to accuracy of computed profile
    - calibrate whenever data is available
  - Contraction and expansion coefficients for X-Sections
    - due to losses associated with changes in X-Section areas and velocities
    - contraction when velocity increases downstream
    - expansion when velocity decreases downstream
  - Bridge and culvert contraction & expansion loss coefficients
    - same as for X-Sections but usually larger values

- Friction loss is evaluated as the product of the friction slope and the discharge weighted reach length

$$h_e = L\overline{S_f} + C\left| \frac{\alpha V_2^2}{2g} - \frac{\alpha V_1^2}{2g} \right|$$

$$L = \frac{L_{lob}\overline{Q_{lob}} + L_{ch}\overline{Q_{ch}} + L_{rob}\overline{Q_{rob}}}{\overline{Q_{lob}} + \overline{Q_{ch}} + \overline{Q_{rob}}}$$

Friction loss is evaluated as the product of the <u>friction slope</u> and the discharge weighted reach length

$$h_e = L\overline{S_f} + C\left| \frac{\alpha V_2^2}{2g} - \frac{\alpha V_1^2}{2g} \right|$$

$$Q = \frac{1.49}{n} A R^{2/3} S_f^{1/2} = K S_f^{1/2}$$

$$S_f^{1/2} = Q/K, \quad S_f = (Q/K)^2$$

Channel Conveyance

Friction Slopes in HEC-RAS

Average Conveyance (HEC-RAS default) - best results for all profile types (M1, M2, etc.)

$$\bar{S}_f = \left(\frac{Q_1 + Q_2}{K_1 + K_2}\right)^2$$

Average Friction Slope - best results for M1 profiles

$$\bar{S}_f = \frac{S_{f_1} + S_{f_2}}{2}$$

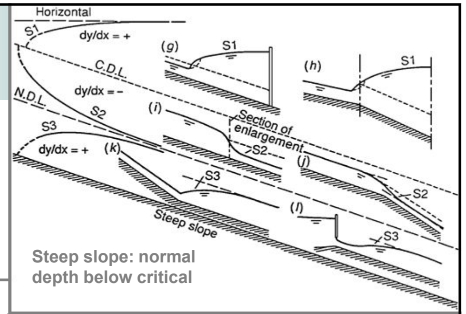
Geometric Mean Friction Slope - used in USGS/FHWA WSPRO model

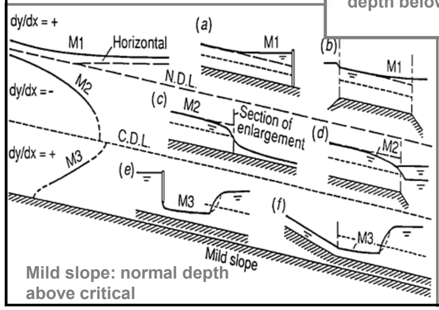
$$\bar{S}_f = \sqrt{S_{f_1} \cdot S_{f_2}}$$

Harmonic Mean Friction Slope - best results for M2 profiles

$$\bar{S}_f = \frac{2S_{f_1} \cdot S_{f_2}}{S_{f_1} + S_{f_2}}$$

Flow Classification





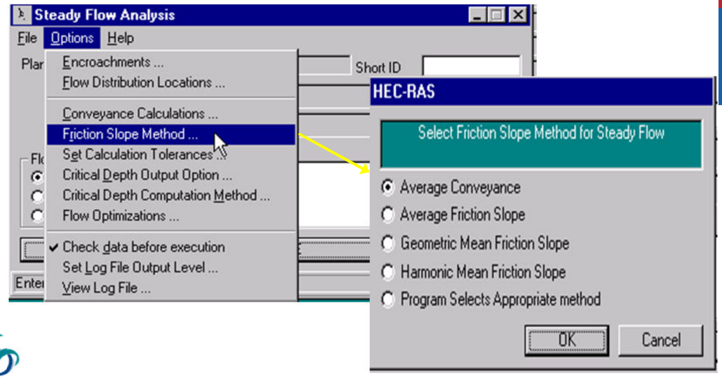
Friction Slopes in HEC-RAS

HEC-RAS has option to allow the program to select best friction slope equation to use based on profile type.

Profile Type	Is friction slope at current cross section greater than friction slope at preceding cross section?	Equation Used
Subcritical (M1, S1)	Yes	Average Friction Slope (2-14)
Subcritical (M2)	No	Harmonic Mean (2-16)
Supercritical (S2)	Yes	Average Friction Slope (2-14)
Supercritical (M3, S3)	No	Geometric Mean (2-15)

Friction Slopes in HEC-RAS

HEC-RAS has option to allow the program to select best friction slope equation to use based on profile type.



HEC-RAS

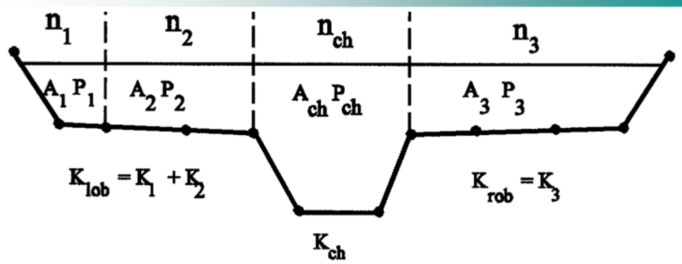


Figure 2.2 HEC-RAS Default Conveyance Subdivision

The default method of conveyance subdivision is by breaks in Manning's 'n' values.

HEC-2 style

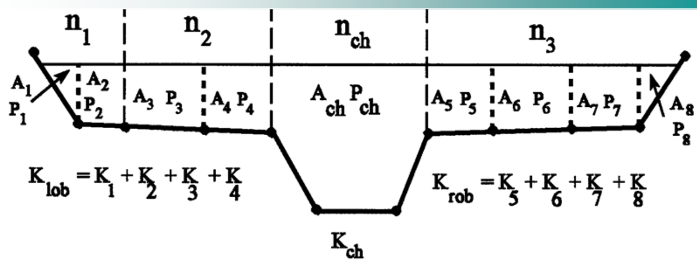


Figure 2.3 Alternative Conveyance Subdivision Method (HEC-2 style)

An optional method is as HEC-2 does it - subdivides the overbank areas at each individual ground point.

Note: can get biggest differences when have big changes in overbanks

Other losses include:

- Contraction losses
- Expansion losses

$$h_e = L\overline{S_f} + C\left| \frac{\alpha V_2^2}{2g} - \frac{\alpha V_1^2}{2g} \right|$$

C = contraction or expansion coefficient

Contraction and Expansion Energy Loss Coefficients

$$h_e = L\overline{S_f} + C\left| \frac{\alpha V_2^2}{2g} - \frac{\alpha V_1^2}{2g} \right|$$

- Note 1: WSP2 uses the upstream section for the whole reach below it while HEC-RAS averages between the two X-sections.
- Note 2: WSP2 only uses LSf in older versions and has added C to its latest version using the "LOSS" card.

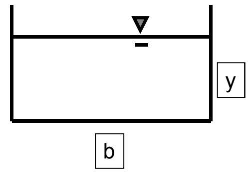
Expansion and Contraction Coefficients

	Expansion	Contraction
No transition loss	0	0
Gradual transitions	0.3	0.1
Typical bridge sections	0.5	0.3
Abrupt transitions	0.8	0.6

Notes: maximum values are 1. Losses due to expansion are usually much greater than contraction. Losses from short abrupt transitions are larger than those from gradual changes.

Specific Energy

Definition: available energy of the flow with respect to the bottom of the channel rather than in respect to a datum.



Assumption that total energy is the same across the section. Therefore we are assuming 1-D.

Note: Hydraulic depth (y) is cross section area divided by top width

$$E = y + \frac{V^2}{2g}$$

$$q = Q/b$$

$$V = Q/by$$

$$V = q/y$$

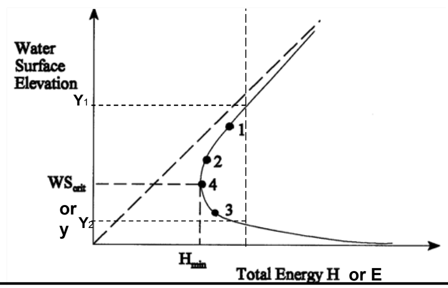
$$E = y + \frac{q^2}{2gy^2}$$

Specific Energy

$$(E-y)y^2 = \frac{q^2}{2g} = const.$$

$$y^2 = \frac{const.}{E-y}$$

The Specific Energy equation can be used to produce a curve.
Q: What use is it?
A: It is useful when interpreting certain aspect of open channel flow



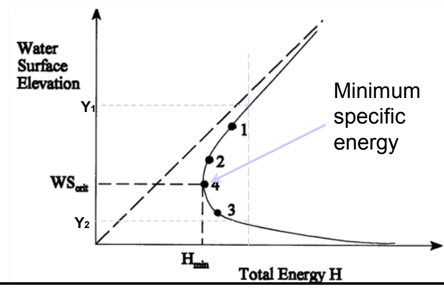
Note: Angle is 45 degrees for small slopes

Specific Energy

$$(E-y)y^2 = \frac{q^2}{2g} = const.$$

$$y^2 = \frac{const.}{E-y}$$

For any pair of E and q, we have two possible depths that have the same specific energy. One is supercritical, one is subcritical. The curve has a single depth at a minimum specific energy.



Specific Energy

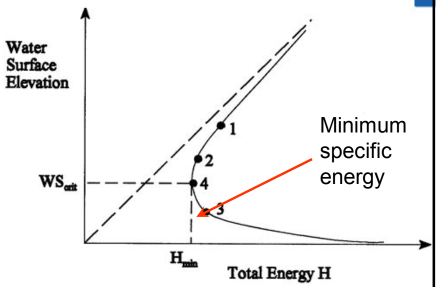
$$E = y + \frac{q^2}{2gy^2}$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 0$$

$$\frac{q^2}{gy^3} = 1$$

Q: What is that minimum?

A: Critical flow!!



Froude Number

$$E = y + \frac{q^2}{2gy^2}$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 1 - Froude^2$$

$$Froude = \frac{q}{\sqrt{gy^3}} = \frac{V}{\sqrt{gy}}$$

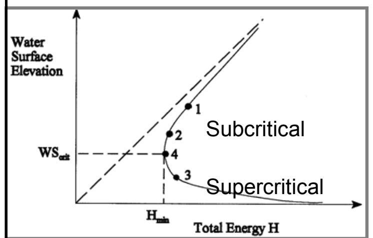
- Ratio of stream velocity (inertia force) to wave velocity (gravity force)

Froude Number

- Ratio of stream velocity (inertia force) to wave velocity (gravity force)

$$Fr = \frac{inertia}{gravity}$$

Fr > 1, supercritical flow
Fr < 1, subcritical flow

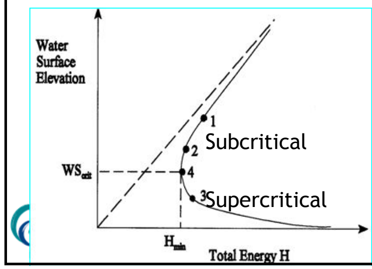


1: subcritical, deep, slow flow, disturbances only propagate upstream

3: supercritical, fast, shallow flow, disturbance can not propagate upstream

Critical Flow

- Froude = 1
- Minimum specific energy
- Transition
- Small changes in energy (roughness, shape, etc) cause big changes in depth
- Occurs at overfall/spillway



$$V_c = \sqrt{gy_c}$$

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$

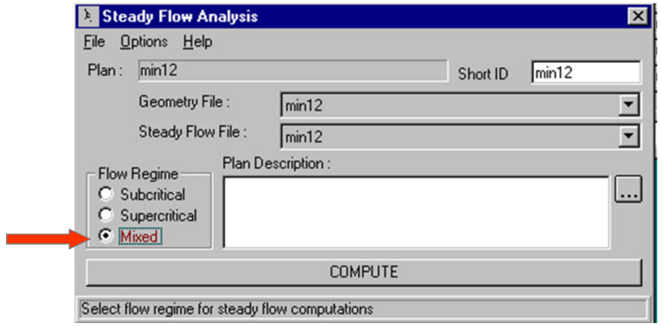
Note: Critical depth is independent of roughness and slope

Critical Depth Determination

HEC-RAS computes critical depth at a x-section under 5 different situations:

- Supercritical flow regime has been specified.
- Calculation of critical depth requested by user.
- Critical depth is determined at all boundary x-sections.
- Froude number check indicates critical depth needs to be determined to verify flow regime associated with balanced elevation.
- Program could not balance the energy equation within the specified tolerance before reaching the maximum number of iterations.

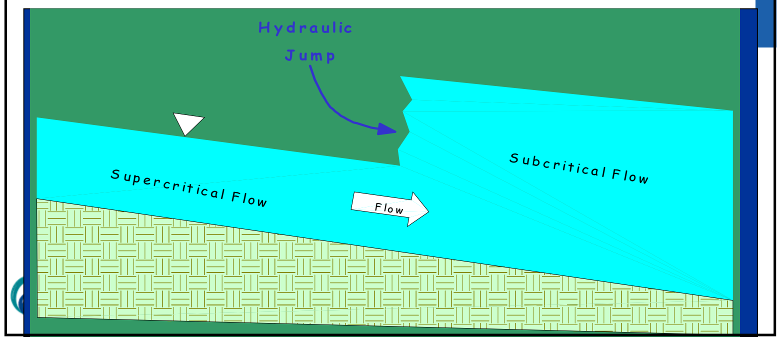
In HEC-RAS, we have a choice for the calculations



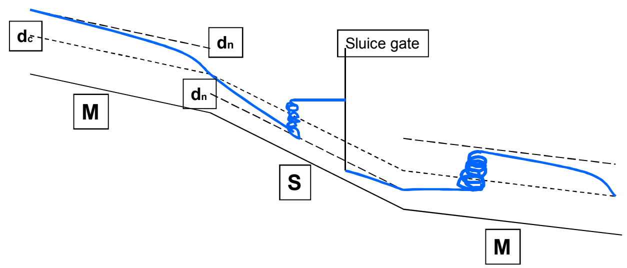
Still need to examine transitions closely

How about hydraulic jumps?

- Water surface “jumps” up
- Typical below dams or obstructions
- Very high-energy loss/dissipation in the turbulence of the jump

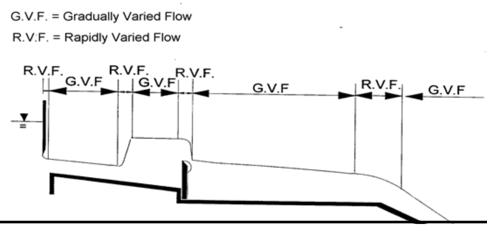


General Shape Of Profile



A rapidly varying flow situation

- Going from subcritical to supercritical flow, or vice-versa is considered a rapidly varying flow situation.
- Energy equation is for gradually varied flow (would need to quantify internal energy losses)
- Can use empirical equations
- Can use momentum equation

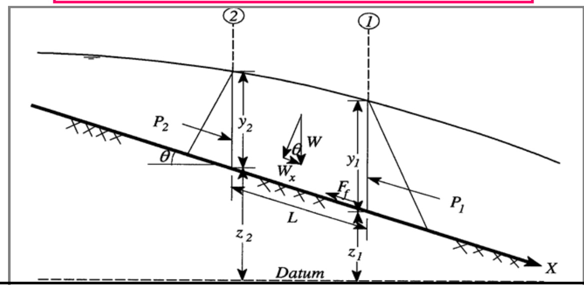


Momentum Equation

- Derived from Newton's second law, F=ma
- Apply F = ma to the body of water enclosed by the upstream and downstream x-sections.

Difference in pressure + weight of water - external friction = mass x acceleration

$$P_2 - P_1 + W_x - F_f = Q \cdot \rho \cdot \Delta V_x$$



Momentum Equation

$$(V_2^2 / 2g) + Y_2 + Z_2 = (V_1^2 / 2g) + Y_1 + Z_1 + hm$$

The momentum and energy equations may be written similarly. Note that the loss term in the energy equation represents internal energy losses while the loss in the momentum equation (hm) represents losses due to external forces.

In uniform flow, the internal and external losses are identical. In gradually varied flow, they are close.

HECRAS can use the momentum equation for:

- Hydraulic jumps
- Hydraulic drops
- Low flow hydraulics at bridges
- Stream junctions.

Since the transition is short, the external energy losses (due to friction) are assumed to be zero

Classifications of Open Steady versus Unsteady

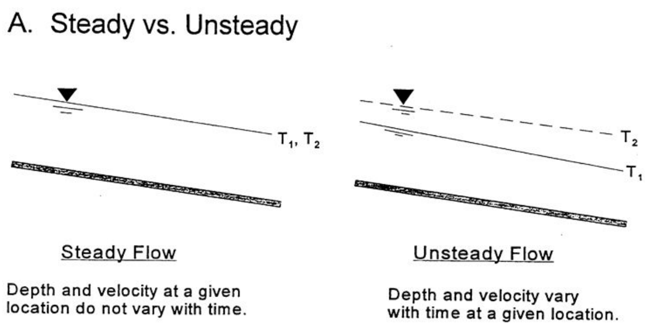


Figure 1 Steady versus Unsteady Flow

Unsteady Examples

Natural streams are always unsteady - when can the unsteady component not be ignored?

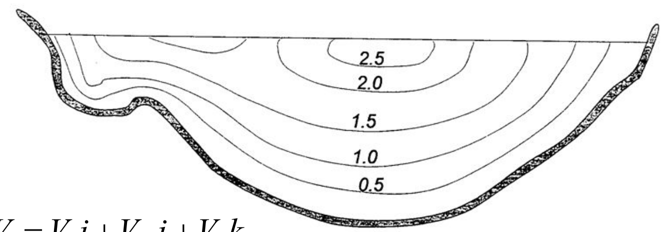
- Dam breach
- Estuaries
- Bays
- Flood wave
- others...

HEC-RAS is a 1-Dimensional Model

- Flow in one direction
- It is a simplification of a chaotic system
- Can not reflect a super elevation in a bend
- Can not reflect secondary currents

Velocity Distribution - It's 3-D

Because of free surface and friction, velocity is not uniformly distributed



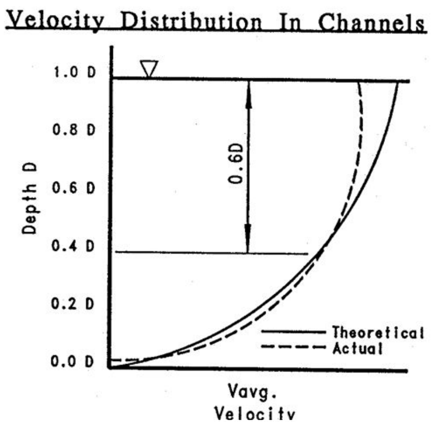
$$V = V_x i + V_y j + V_z k$$

Figure 4 Example 2-D Velocity Distribution

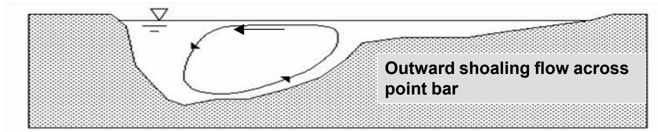
...but HEC-RAS is 1-D

Velocity Distribution

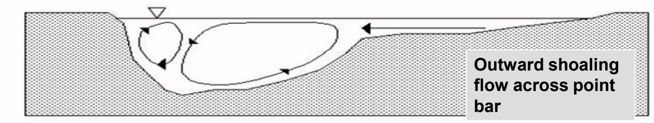
- Actual Max V is at approx. 0.15D (from the top).
- Actual Average V is at approx.. 0.6D (from the top).



Secondary circulation pattern at a river bed cross section



Single cell theory (Thomson, 1876; Hawthorne, 1951; Quick, 1974)



Current theory of bend flow with skew induced outer bank cells (Hey and Thorne, 1975)

Other assumptions in HEC-RAS

- HEC-RAS is a fixed bed model
  - The cross section is static
  - HEC-6 allows for changes in the bed.
- HEC-RAS can not by itself reflect upstream watershed changes.

HEC-RAS is a simplification of the natural system

End of lecture